MA114 Summer 2018
Worksheet 4 – Trig Substitutions 6/13/18

1. Compute the following integrals.

a) $\int x\sqrt{1-4x^2}\,dx$, $\quad u = 1-4x^2 \quad du = -8x\,dx$

$$= -\frac{1}{8}\int u^{1/2}\,du = -\frac{1}{8}\cdot\frac{2}{3}(1-4x^2)^{3/2} + C$$

$$= -\frac{1}{12}(1-4x^2)^{3/2} + C$$

b) $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$, Hint: Complete the square.

$$3-(2x+x^2) = 3-(x^2+2x+1-1) = 3-(x^2+2x+1)+1 = 4-(x+1)^2$$

$$= \int \frac{x}{\sqrt{4-(x+1)^2}}\,dx \qquad u = x+1$$

$$= \int \frac{u-1}{\sqrt{4-u^2}}\,du \qquad u = 2\sin\theta \quad du = 2\cos\theta\,d\theta$$

$$= \int \frac{-1+2\sin\theta}{2\cos\theta}\,2\cos\theta\,d\theta$$

$$= \int -1+2\sin\theta\,d\theta = -\theta - 2\cos\theta + C = -\sin^{-1}\left(\frac{u}{2}\right) - \sqrt{4-u^2} + C$$

$$= -\sin^{-1}\left(\frac{x+1}{2}\right) - \sqrt{3-2x-x^2} + C$$

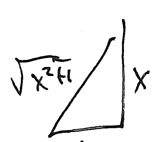
c) $\int \frac{x^2\,dx}{(x^2+1)^{3/2}}$. $\quad = \int \frac{\tan^2\theta\sec^2\theta\,d\theta}{(\tan^2\theta+1)^{3/2}} = \int \frac{\tan^2\theta\sec^2\theta\,d\theta}{\sec^3\theta}$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta$$

$$= \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$



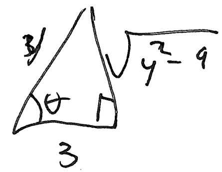
$$= \int \sec\theta - \cos\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| - \sin\theta + C$$

$$= \ln\left|\sqrt{x^2+1} + x\right| - \frac{x}{\sqrt{x^2+1}} + C$$

d) $\displaystyle\int \frac{dy}{\sqrt{y^2-9}}$.

$y = 3\sec\theta$
$dy = 3\sec\theta\tan\theta\, d\theta$



$$= \int \frac{3\sec\theta\tan\theta\, d\theta}{\sqrt{9\sec^2\theta - 9}}$$

$$= \int \frac{3\sec\theta\tan\theta\, d\theta}{3\tan\theta}$$

$$= \int \sec\theta\, d\theta$$

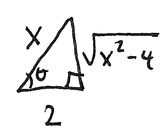
$$= \ln|\sec\theta + \tan\theta| + C = \underline{\ln\left|\frac{y}{3} + \frac{\sqrt{y^2-9}}{3}\right| + C}$$

2. Is the substitution $u = x^2 - 4$ effective for evaluating the integral $\displaystyle\int \frac{x^2\,dx}{\sqrt{x^2-4}}$? If not, evaluate using trigonometric substitution.

No. $x = 2\sec\theta$
$dx = 2\sec\theta\tan\theta\, d\theta$

$$\int \frac{x^2\,dx}{\sqrt{x^2-4}} = \int \frac{4\sec^2\theta \cdot 2\sec\theta\tan\theta\, d\theta}{\sqrt{4\sec^2\theta - 4}}$$

$$= \int \frac{8\sec^3\theta\ \tan\theta\, d\theta}{2\tan\theta}$$



$$= 4\int \sec^3\theta\, d\theta$$

see pg 2 of posted notes on 7.2

$$= 2(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|) + C$$

$$= 2\left(\frac{x}{2}\cdot\frac{\sqrt{x^2-4}}{2} + \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right|\right) + C$$

$$= \underline{\frac{1}{2}x\sqrt{x^2-4} + 2\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C}$$